E 4690 (Pages : 4) Reg. No.....

Name.....

B.Sc. DEGREE (C.B.C.S.S.) EXAMINATION, NOVEMBER 2012

First Semester

Core Course—FOUNDATION OF MATHEMATICS

(Common for Model I and Model II B.Sc. Mathematics and B.Sc. Computer Applications)

Time : Three Hours Maximum Weight : 25

Part A (Objective Type Questions)

Answer **all** *questions.*

A bunch of **four** *questions has weight* 1.

I. 1. Find $A \times B \times C$ where

$A = \{0, 1\}$, $B = \{1, 2\}$ and $C = \{0, 1, 2\}$.

2. Let f be a function from the set A to the set B. Let S and T be subsets of A. Show that :

$$f(S \cap T) \subseteq f(S) \cap f(T).$$

3. Let x be a real number with $|x| < 1$. Find $\sum_{n=0}^{\infty} x^n$.

4. Let R be the relation :

$\{(1, 2), (1, 3), (2, 3), (2, 4), (3, 1)\}$

and let S be the relation :

$\{(2, 1), (3, 1), (3, 2), (4, 2)\}$. Find $S \circ R$?

II. 5. Define the term "The directed graph".

6. What are the equivalence classes of 0 and 1 for Congruence modulo 4.

7. Show that the inclusion relation '$\subseteq$' is a partial ordering on the power set of a set S.

8. Is the Poset $(Z^+, 1)$ a lattice ?

III. 9. Express the specification "The automated reply can not be sent when the file system is full" using logical connectives.

10. Construct a truth table for the Compound proposition $(p \vee q) \oplus (p \wedge q)$.

Turn over

11. Prove that the sum of two rational numbers is rational.

12. Show that at least four of any 22 days must fall on the same day of the week.

IV. 13. Prove that $3^{2n+1}+2^{n+2}$ is divisible by 7.

14. Write a short note on the Euler's generalisation of the Fermat's theorem.

15. Find x such that $36x \equiv 7 \pmod{157}$.

16. Show that L18 + 1 is divisible by 437.

$(4 \times 1 = 4)$

Part B *(Short Answer Questions)*

Answer any **five** *questions.*
Each question has weight 1.

17. If A and B are two sets, prove that $A \oplus B = B \oplus A$.

18. Find the matrix representation $S \circ R$ where the matrices representing R and S are

$$M_R = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \text{ and } M_S = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 1 \end{bmatrix}.$$

19. Let R be the relation on the set of ordered pairs of positive integers such that $((a, b), (c, d)) \in R$ if and only if $ad = bc$. Show that R is an equivalence relation.

20. Let p and q be the propositions "The election is decided" and "The votes have been counted" respectively. Express the Compound proposition

$\neg q \vee (\neg p \wedge q)$ as an English Sentence.

21. Compare the terms Tautology and Contradiction.

22. Show that $2^{2n} - 3n - 1$ is divisible by 9.

23. With usual notations, prove that $\phi(mn) = \phi(m) \cdot \phi(n)$.

24. Explain how to find x such that

$x \equiv a \pmod{\alpha}$ and
$x \equiv b \pmod{\beta}$.

(5 × 1 = 5)

Part C (Short Essay Questions)

Answer any **four** *questions.*
Each question has weight 2.

25. Prove that there are no solutions in integers x and y to the equation $2x^2 + 5y^2 = 14$.

26. How many reflexive relations are there on a set with n elements ?

27. Show that $(\neg q \vee q)$ and $\neg p \wedge \neg q$ are logically equivalent.

28. Translate the statement $\exists x \, \forall y \, \forall z \left(F(x, y) \wedge F(x, z) \wedge (y \neq z) \rightarrow \neg F(y, z) \right)$ into English where $F(a, b)$ means that a and b are friends and the universe of discourse for x, y and z consists of all students in the School.

29. Explain the method of Mathematical Induction. Use it to prove that

$$\frac{1}{n} + \frac{1}{n+1} + \frac{1}{n+2} + \ldots + \frac{1}{2n-1} = 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \ldots + \frac{1}{2n-1}$$

where n is a positive integer.

30. Express $\frac{101}{1540}$ in the form $\frac{x}{4} + \frac{y}{5} + \frac{z}{7} + \frac{u}{11} + k$, the fractions being Positive proper fractions and k an integer.

(4 × 2 = 8)

Part D (Essay Questions)

Answer any **two** *questions.*
Each question has weight 4.

31. Let R be the relation represented by the matrix :

$$M_R = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}.$$

Find the matrix representing :

(a) R^{-1}.

(b) $\overline{R}$.

(c) R^2.

32. Show that the propositions $p \vee (q \wedge r)$ and $(p \vee q) \wedge (p \vee r)$ are logically equivalent.

33. (a) State and prove the Wilson's Theorem.

(b) Prove that L28 + 233 is divisible by 899.

$(2 \times 4 = 8)$